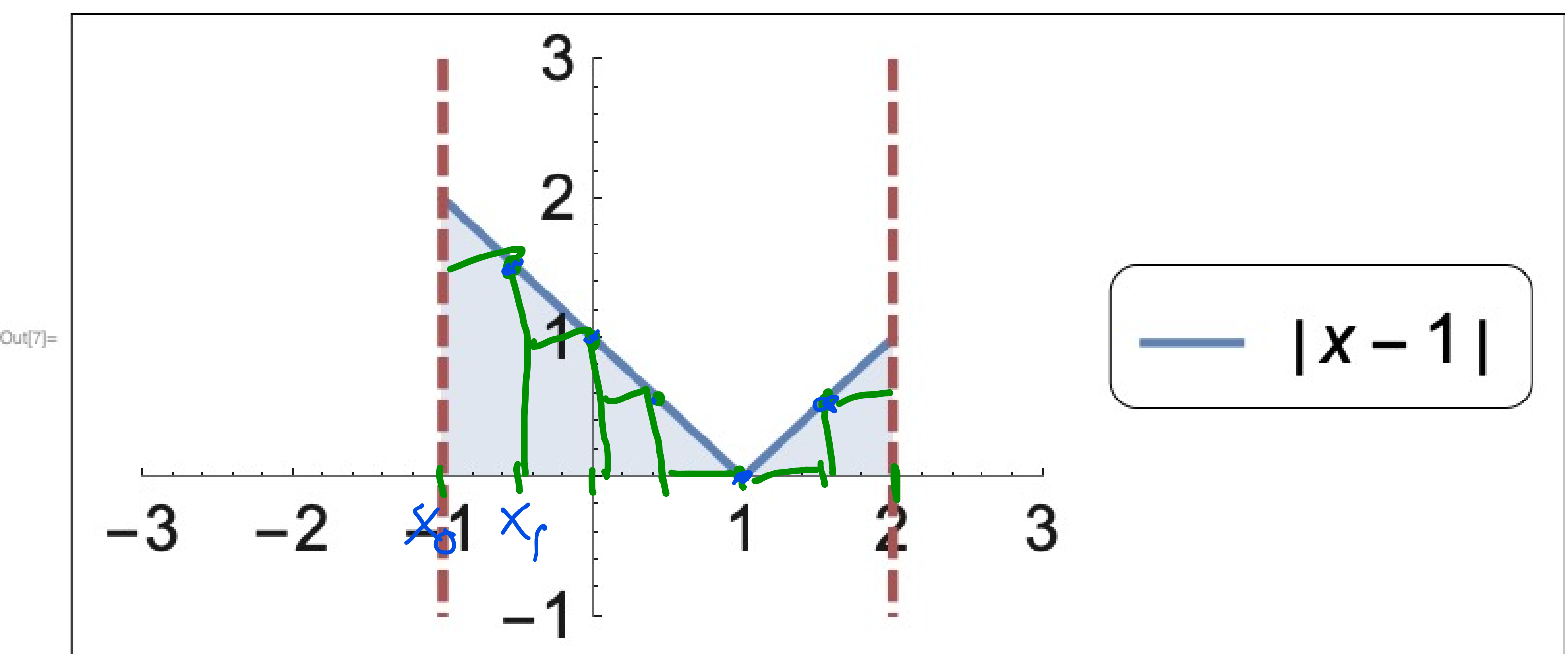
Q10: (+6 points): Numerically approximate

$$I = \int_{-1}^{2} 24|x-1|\,dx$$ with

N = 6 rectangles.

Find the lower sum estimate (L)

$$\Delta x = \frac{2-(-1)}{6} = 1/2$$



$x_0 = -1,\ x_1 = -1/2,\ x_2 = 0,\ x_3 = 1/2,$

$x_4 = 1,\ x_5 = 3/2,\ x_6 = 2$

- for $x \in [-1,1]$, the rectangle heights are the function values at the RHS endpoints,

and for $x \in [1,2]$ at the LHS endpoints.

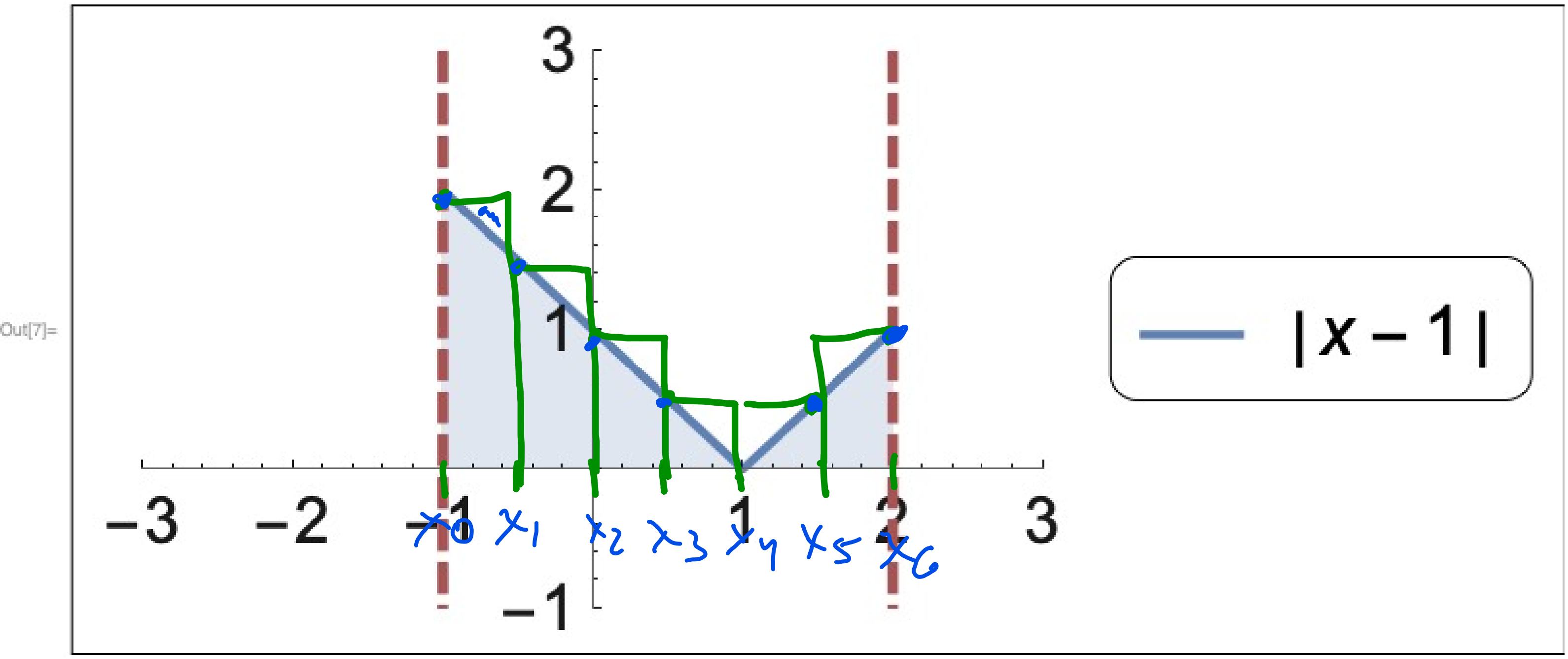
- So for $f(x) = |x-1|$,

$$L = 24\Delta x\left(f(-1/2) + f(0) + f(1/2) + f(1) + f(1) + f(3/2)\right)$$

$$= 12 \cdot \left(\frac{3}{2} + 1 + \frac{1}{2} + 0 + 0 + \frac{1}{2}\right)$$

$$= 12 \cdot \frac{7}{2} = 6 \cdot 7$$

$$= 42$$



Similarly, to find U:

$$U = 24\Delta x\left(2+\frac{3}{2}+1+\frac{1}{2}+\frac{1}{2}+1\right)$$

$$= 12\cdot\frac{13}{2} = 6\cdot 13 = 78$$

Final Review:

$$I = \int_0^{\left(\frac{\pi}{2}\right)^{1/7}} x^6 \cdot \sin(x^7)\,dx$$

u-sub: $u = x^7$, $du = 7 \cdot x^6\,dx$,

$u(0) = 0^7 = 0$,

$u\left(\left(\frac{\pi}{2}\right)^{1/7}\right) = \pi/2 = \left(\left(\frac{\pi}{2}\right)^{1/7}\right)^7$

$$= \frac{1}{7}\int_0^{\pi/2} \sin(u)\,du$$

$$= -\frac{1}{7}\cos(u)\Big|_0^{\pi/2}$$

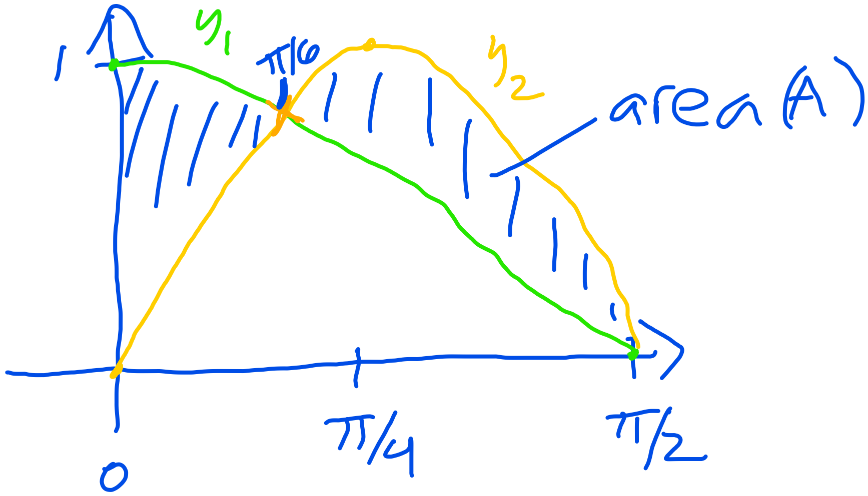
$$= -\frac{1}{7}\left(\cos(\pi/2) - \cos(0)\right) = \frac{1}{7}$$

#20 on the midterm review:

find the area bounded by

$y_1 = \cos(x)$

$y_2 = \sin(2x)$, for $0 \leq x \leq \pi/2$



→ find the intersection point ($y_1 = y_2$)

$$\cos(x) = \sin(2x)$$
$$= 2 \cdot \sin(x) \cdot \cos(x)$$

$$\Longleftrightarrow \sin(x) = \frac{1}{2} \Longleftrightarrow x = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

→ Compute the area:

$$A = \int_0^{\pi/6} \left(y_1(x) - y_2(x)\right) dx + \int_{\pi/6}^{\pi/2} \left(y_2(x) - y_1(x)\right) dx$$

$$= \int_0^{\pi/6} \left(\cos(x) - \sin(2x)\right) dx$$

$$+ \int_{\pi/6}^{\pi/2} \left(\sin(2x) - \cos(x)\right) dx$$

$$= \left(\sin(x) + \frac{1}{2}\cos(2x)\right)\Bigg|_0^{\pi/6}$$

$$+\left(-\frac{1}{2}\cos(2x)-\sin(x)\right)\Big|_{\pi/6}^{\pi/2}$$

$$=\sin(\pi/6)+\frac{1}{2}\cos(\pi/3)$$

$$-\left(\sin(0)+\frac{1}{2}\cos(0)\right)$$

$$+\left(-\frac{1}{2}\cos(\pi)-\sin(\pi/2)\right)$$

$$+\left(\frac{1}{2}\cos\left(\frac{\pi}{3}\right)+\sin(\pi/6)\right)$$

$$=\frac{1}{2}+\frac{1}{4}-\left(0+\frac{1}{2}\right)$$

$$+\left(\frac{1}{2}-1\right)+\left(\frac{1}{4}+\frac{1}{2}\right)=1/2$$

#25(d) on the midterm review:

$$I=\int x\cdot\underline{\sin(x)\cos(x)}\,dx$$

$$\sin(2x)=2\sin(x)\cos(x)$$

$$=\frac{1}{2}\int x\cdot\sin(2x)\,dx$$

IBP: • $\int u\,dv = uv - \int v\,du$

• ILATE

• $u = x$ $\qquad dv = \sin(2x)\,dx$

$du = dx$ $\qquad v = -\frac{1}{2}\cos(2x)$

$$= \frac{1}{2}\left[-\frac{1}{2}x\cos(2x) + \frac{1}{2}\int \cos(2x)\,dx\right]$$

$$= -\frac{1}{4}x\cos(2x) + \frac{1}{8}\sin(2x) + C$$

#4(a) and (c) under studio problems from the midterm review:

$$L = \lim_{x\to 0^+} x\cdot[\ln(x)]^2$$

$$= \lim_{x\to 0^+} \frac{[\ln(x)]^2}{\frac{1}{x}}$$

$\frac{\infty}{\infty} \rightarrow$ apply L'Hopital's rule

$$= \lim_{x\to 0^+} \frac{2\cdot\ln(x)\cdot\frac{1}{x}}{-\frac{1}{x^2}}$$

$$= \lim_{x\to 0^+} \frac{-2\ln(x)}{1/x}$$

$\frac{\infty}{\infty} \rightarrow$ again

$$= \lim_{x\to 0^+} \frac{-\frac{2}{x}}{-\frac{1}{x^2}} = \lim_{x\to 0^+} 2x = 0$$

$$L = \lim_{x\to\pi/2} \frac{\ln(\sin(x))}{(\pi-2x)^2} \quad \frac{0}{0}$$ → apply L'Hopital's rule

$$= \lim_{x\to\pi/2} \frac{\frac{1}{\sin(x)}\cdot\cos(x)}{2(\pi-2x)(-2)}$$] num] denom $\frac{0}{0}$ → again

$$= \lim_{x\to\pi/2} \frac{-\csc^2(x)}{2(-2)(-2)}$$

$$= -\frac{1}{8}\lim_{x\to\pi/2} \frac{1}{\sin^2(x)} = -\frac{1}{8}$$

#3(f) under studio problems on the midterm review:

$$I = \int \frac{x+3}{(x-1)(x^2-4x+4)}\,dx$$ → apply partial fractions

$$= \int \frac{x+3}{(x-1)(x-2)^2}\,dx$$

→ write out the partial fractions decomposition:

$$\frac{x+3}{(x-1)(x-2)^2} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{(x-2)^2} \quad (*)$$

$$x+3 = A(x-2)^2 + B(x-1)(x-2) + C(x-1)$$

- when $x=2$:

$$5 = C$$

- when $x=1$:

$$4 = A$$

- when $x=0$:

$$3 = 4A + 2B - C$$

$$3 = 11 + 2B \longleftrightarrow 2B = -8$$

$$\longleftrightarrow B = -4$$

→ Now integrate $(*)$:

$$I = \int \frac{4dx}{x-1} - 4\int \frac{dx}{x-2} + 5\int \frac{dx}{(x-2)^2}$$

$$= 4\ln|x-1| - 4\ln|x-2|$$

$$-\frac{5}{(x-2)} + C$$